

Expressions and Equations

2

Unit Overview

In this unit, you will create and solve linear equations and inequalities from tables, graphs, and verbal descriptions. You will represent equations and inequalities on number lines.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- palindrome
- media

Math Terms

- property
- numerical statement
- algebraic expression
- coefficient
- equation
- numerical expression
- variable
- algebraic statement
- constant

ESSENTIAL QUESTIONS



Why is it important to understand how to solve linear equations and inequalities?



How can graphs be used to interpret solutions of real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 6 and 7, will give you an opportunity to demonstrate how you will use multiple representations to write and solve linear equations and inequalities.

Embedded Assessment 1:

Writing and Solving Equations p. 65

Embedded Assessment 2:

Solving Inequalities p. 75

Getting Ready

Write your answers on notebook paper.
Show your work.

1. A car travels 50 miles per hour.
a. Complete the table below to show the total distance traveled for each time given.

Number of hours that have passed	Total distance traveled
1	
2	
3	

- b. Plot the data from the table.
c. If the car has traveled n hours, write an expression for the total distance traveled.
d. How far has the car traveled after 10 hours? Explain how you determined your answer.
2. Solve each equation below.
a. $3x = 12$
b. $x + 5 = -4$
c. $2x - 5 = 7$
3. Give 3 examples of integers which are
a. greater than -2
b. less than or equal to 1
4. Tell 3 numbers that are less than 2 and greater than -1 .
5. Evaluate each of the following
a. 2^3 b. 3^2 c. $3 + 4 \times 2$ d. $2 \times 3 + 4$
6. Write an algebraic expression to represent each of the following.
a. a number increased by 7
b. 8 times a number
c. 6 less than 3 times a number
7. Two measures of two angles of a triangle are 68° and 70° . Explain how to find the measure of the third angle.
8. The Harris family is planning to buy a new 46-inch HDTV that costs \$488. Mr. and Mrs. Harris will pay \$200 and their three sons will split the remaining cost equally. Explain how to find the amount each of the boys will pay.

Properties of Operations

What's in a Name?

Lesson 5-1 Applying Properties of Operations

Learning Targets:

- Identify properties of operations.
- Apply properties of operations to simplify linear expressions.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Work Backward, Think-Pair-Share, Note Taking, Sharing and Responding

Words and numbers that have the property of being read the same forward and backward are called **palindromes**. The names Hannah and Otto are two examples of names that are palindromes.

1. What other names or words do you know that are palindromes?

Numbers and operations have interesting **properties** as well.

2. Determine if each statement below is true or false.
 - a. $3 + 5 = 5 + 3$
 - b. $3 \cdot 5 = 5 \cdot 3$
 - c. $2 + (-2) = 0$
 - d. $10 \div 2 = 2 \div 10$
 - e. $6 - 3 = 3 - 6$
 - f. $2(5 \cdot 3) = (2 \cdot 5)3$
 - g. $5 + 0 = 0$
 - h. $2\left(\frac{1}{2}\right) = 0$
 - i. $2 + (4 + 5) = (2 + 4) + 5$
 - j. $1 \cdot 3 = 3$
3. Which of the true statements above are similar to a palindrome?

Some properties of operations are listed below. As you share ideas about the information given in the table and throughout this lesson, ask your group members or your teacher for clarification of any language, terms, or concepts that you do not understand.

Property	Example
Additive Identity Property	$12 + 0 = 0 + 12 = 12$
Multiplicative Identity Property	$5 \times 1 = 1 \times 5 = 5$
Commutative Property of Addition	$7 + 3 = 3 + 7$
Commutative Property of Multiplication	$20 \times 4 = 4 \times 20$
Associative Property of Addition	$(9 + 4) + 17 = 9 + (4 + 17)$
Associative Property of Multiplication	$(10 \times 5) \times 3 = 10 \times (5 \times 3)$
Additive Inverse Property	$6 + (-6) = 0$
Multiplicative Inverse Property	$4\left(\frac{1}{4}\right) = 1$

My Notes

ACADEMIC VOCABULARY

A **palindrome** is a word, phrase, or sequence that reads the same backward or forward.

MATH TERMS

A **property** is a rule or statement that is always true.

A **numerical expression** is an expression that contains numbers and operations. For example, $12 + 0$ and $(10 \times 5) \times 3$ expression that contains numbers and operations.

A **numerical statement** is an equation that sets two numerical expressions equal. For example, $20 \times 4 = 4 \times 20$.

My Notes

4. **Reason abstractly.** The first seven properties listed in the table on the previous page are true for all numbers. The Multiplicative Inverse Property is true for all numbers except 0. Why does 0 not have a multiplicative inverse?
5. **Reason abstractly.** Do the commutative and associative properties apply for the operations of subtraction and division? Justify your response with numeric examples.
6. The number 0 is called the additive identity element because when you add 0 to a given number, the identical given number is the result. Explain why 0 is not the multiplicative identity element.
7. A hiker walked 2.75 miles directly east and then walked 2.75 miles directly west. Use the Additive Inverse Property to explain why he ends up back where he started.

You can illustrate the properties using algebraic expressions and equations.

Example A

Write an algebraic statement of the Commutative Property of Multiplication.

Step 1: Choose a variable to represent the first number.

Let a = the first number

Step 2: Choose a variable to represent the second number.

Let b = the second number

Step 3: Write an algebraic statement of the property.

$$a \cdot b = b \cdot a$$

Solution: An algebraic statement of the Commutative Property of Multiplication is $a \cdot b = b \cdot a$

Try These A

State the property illustrated by each algebraic statement.

a. $a + b = b + a$

b. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

c. $a \cdot 1 = a$

d. $a + 0 = a$

e. $a \frac{1}{a} = 1; a \neq 0$

VOCABULARY MATH TERMS

A **variable** is a letter used in place of a number.

MATH TERMS

An **algebraic expression** is an expression that contains one or more variables, such as $4a + 7$.

An **algebraic statement** is an equation or inequality that contains variables, such as $(a + b) + c = a + (b + c)$.

Check Your Understanding

8. Explain how you can use properties of operations as strategies to evaluate each expression using mental math. Then evaluate the expression.
 - a. $0.6 + 12.7 + 1.4 + 2.3$
 - b. $4.5 + 3.9 + (-0.5) + (-4)$
 - c. $8\frac{3}{5} - 9\frac{1}{2} + 1\frac{2}{5}$
 - d. $4\frac{5}{8} + 6\frac{7}{8} - 3\frac{5}{8}$
9. Name the property of operations illustrated by each statement.
 - a. $x \cdot 4y = 4x \cdot y$
 - b. $100 \cdot 1 = 100$
 - c. $x(3 \cdot 2x) = (x \cdot 3) \cdot 2x$
 - d. $3 + (-3) = 0$
 - e. $-8\left(-\frac{1}{8}\right) = 1$

LESSON 5-1 PRACTICE

10. What is the value of n in $(17 \times 25) \times 4 = n \times (25 \times 4)$?
11. Name the property of operations illustrated by each statement.
 - a. $11.1 + 0 = 11.1$
 - b. $6(2.7 + 4) = 6(4 + 2.7)$
 - c. $\left(\frac{1}{2} \cdot 3\right) \cdot 1\frac{2}{3} = \frac{1}{2} \cdot \left(3 \cdot 1\frac{2}{3}\right)$
 - d. $18 + (-18) = 0$
12. Write an algebraic statement to illustrate each of the following properties. Use x , y , and z to represent the numbers.
 - a. Multiplicative Identity Property
 - b. Associative Property of Addition
 - c. Commutative Property of Multiplication
 - d. Additive Inverse Property
 - e. Additive Identity Property
 - f. Multiplicative Inverse Property
 - g. Commutative Property of Addition
 - h. Associative Property of Multiplication

My Notes

My Notes

DISCUSSION GROUP TIPS

The word *factor* when used as a verb means to write a number or expression as a product of its factors.

The word *factor* when used as a noun means any of the numbers that are multiplied together to form a product.

As you work through this lesson with your partner or group, look for and identify both uses of factor.

WRITING MATH

A multiplication expression can be written in two ways: $5 \cdot n$ or $5n$.

Learning Targets:

- Apply properties to factor and expand linear expressions.
- Rewrite expressions to see how the problem and quantities are related.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Interactive Word Wall, Summarizing, Think-Pair-Share, Quickwrite

The *Distributive Property* can be used to expand or factor an expression.

Distributive Property of Multiplication over Addition:

- To expand an expression:
 $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$
- To factor an expression:
 $ab + ac = a(b + c)$ or $ba + ca = (b + c)a$

Distributive Property of Multiplication over Subtraction:

- To expand an expression:
 $a(b - c) = ab - ac$ or $(b - c)a = ba - ca$
- To factor an expression:
 $ab - ac = a(b - c)$ or $ba - ca = (b - c)a$

Example A

Use the Distributive Property to expand $5(x + 2)$.

Step 1: Multiply 5 by the terms in the parentheses.

$$5(x + 2) = 5 \cdot x + 5 \cdot 2$$

Step 2: Simplify.

$$5 \cdot x + 5 \cdot 2 = 5x + 10$$

Solution: In expanded form, $5(x + 2) = 5x + 10$.

Try These A

Use the Distributive Property to expand each expression.

- a. $4(6 - 2)$
- b. $7(2 + 5)$
- c. $6(a + 7)$
- d. $3(4 - b)$

Example B

Use the Distributive Property to factor $12x - 18$.

Step 1: Determine the greatest common factor of each term.

$$12x - 18$$

The greatest common factor of $12x$ and 18 is 6 .

Step 2: Divide by the common factor.

$$\frac{12x}{6} - \frac{18}{6}$$

Divide both terms by 6 .

Step 3: The greatest common factor, 6 , is factored out of the terms and is shown outside the parentheses with the quotients of the division inside the parentheses.

Rewrite using parentheses.

$$6(2x - 3)$$

Solution: In factored form, $12x - 18 = 6(2x - 3)$.

Try These B

Use the Distributive Property to factor each expression.

a. $12 + 10$

b. $18 - 6$

c. $6x + 3y$

d. $2a - 10$

e. $5x + 5$

f. $12r - 24$

Equivalent expressions are two or more expressions that may look different, but represent the same quantity or have equal values when evaluated. The expression $2x + 4x$ and the expression $6x$ are equivalent expressions.

Example C

Use the Distributive Property to simplify $5a + 3a$.

Factor the expression using the distributive property.

$$5a + 3a = (5 + 3)a = 8a$$

Solution: The expression $5a + 3a$ can be simplified to $8a$.

Try These C

Use the Distributive Property to simplify each expression.

a. $6x + 9x$

b. $5b - 2b$

c. $12d - 8d$

d. $3h + (-7h)$

My Notes

MATH TIP

The **order of operations** is a set of rules for evaluating expressions with more than one operation. The order is as follows:

1. Do calculations inside grouping symbols first, beginning with the innermost set.
2. Evaluate expressions with exponents.
3. Multiply or divide from left to right.
4. Add or subtract from left to right.

MATH TIP

Remember that percent means hundredths, so

$$85\% = \frac{85}{100} = 0.85 \text{ and}$$

$$7\% = \frac{7}{100} = 0.07.$$

1. Write an equivalent expression for $3(y - 6) + 4$.
2. **Construct viable arguments.** Izzi thinks the two expressions $2(2a - 1) + 3a$ and $7a - 2$ are equivalent. His work is shown below. Is he correct? Why or why not? List the properties of operations and the order of operations next to each step to justify your response.

$$2(2a - 1) + 3a$$

$$4a - 2 + 3a$$

$$4a + 3a - 2$$

$$(4 + 3)a - 2$$

$$7a - 2$$

3. Naman and Ada disagree about how to find an equivalent expression for $2(4x - 3) + 6$. Who is correct? How do you know? Use properties of operations and the order of operations to justify your response.

<u>Naman</u>	<u>Ada</u>
$2(4x - 3) + 6$	$2(4x - 3) + 6$
$6 + 2(4x - 3)$	$8x - 6 + 6$
$8(4x - 3)$	$8x + 0$
$32x - 24$	$8x$

Rewriting an expression in a different form can show how quantities are related.

Example D

Use the Distributive Property to show that increasing an amount by 8% is the same as multiplying the amount by 1.08.

Step 1: Choose an amount.
50

Step 2: Add to show the original amount plus the 8% increase.
 $50 + 50(0.08) = 50 + 4 = 54$

Step 3: Find the common factor of 50 and $50(0.08)$.
The common factor of $50 + 50(0.08)$ is 50.

Step 4: Use the Distributive Property to rewrite $50 + 50(0.08)$.
 $50 + 50(0.08) = 50(1 + 0.08) = 50 + 4 = 54$

Solution: Increasing an amount by 8% is the same as multiplying the amount by $(1 + 0.08)$ or 1.08.

Lesson 5-2

Applying Properties to Factor and Expand

ACTIVITY 5

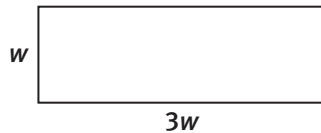
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Try These D

- Louisa wants to buy an online movie subscription that is on sale for 15% off. She writes the expression $c - 0.15c$ to represent the cost of the subscription. Rewrite this expression in a different form to show what percent of the original price she will pay for the online movie subscription. Then compare your expression with Louisa's. How are the expressions related? What does each expression tell you about the problem situation?
- The Rumpart family is building a new room onto their house. The width of the new room will be 16 feet. The length of the room will be 4% greater than the width. Write an expression to find the length of the new room. What will be the area of this new room?

Check Your Understanding

- Use the Distributive Property to write an equivalent expression for each of the following.
 - $(q - 6)7$
 - $3(11 + 5x)$
 - $4x - 16$
 - $(24 + 6x)$
- The length of a rectangle is three times its width. One way to write an expression to find the perimeter would be $w + w + 3w + 3w$. Write the expression in two other different ways.



LESSON 5-2 PRACTICE

- Constructing viable arguments.** Explain how $a(b + c)$ can be rewritten as $(b + c)a$ and as $ba + ca$.
- Use the Distributive Property to write an equivalent expression for each of the following.
 - $4(x - 3)$
 - $12x + 24y$
 - $9 - 9z$
 - $(7y - 1)8$
- The expression $x - 0.10x$ gives the cost of an item that is discounted 10%. Write this expression another way.

My Notes

ACTIVITY 5

continued

ACTIVITY 5 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 5-1

For 1–4, name the property of operations illustrated by each statement.

1. $4 \cdot 1 = 4$
2. $3(xy + z) = (xy + z)3$
3. $7b + 0 = 7b$
4. $9x + 3y + 4x = 9x + 4x + 3y$

For 5–8, complete each statement. Then state the property or properties illustrated by the statement.

5. $23 + \square = 23$
6. $\square \cdot (-5x) = 1$
7. $-15a + \square = 65 + (-15a)$
8. $(8 \cdot \square) \cdot (-77) = -77 \cdot (\square \cdot 8)$

For 9–12, write an algebraic statement that illustrates each property.

9. Associative Property of Addition
10. Commutative Property of Multiplication
11. Additive Identity Property
12. Multiplicative Inverse Property

Lesson 5-2

For 13–15, write an algebraic statement that illustrates each property.

13. Distributive Property of Multiplication over Addition
14. Distributive Property of Multiplication over Subtraction
15. In which properties are more than one operation used? Include an example in your response.

16. Which expression is equivalent to $2a + 13 - 0 + 65b$?
 - A. $2a + 13 - 65b$
 - B. $2a + 65b + 13$
 - C. $(2a + 13) \cdot 65b$
 - D. $67ab + 13$

For 17–19, completely factor each expression.

17. $2x + 12$
18. $24 + 8y - 16w$
19. $14 - 8m$

20. Laura is paying $1.07x$ including tax for a sweater that costs x before tax. Write this expression in another way to show the amount of tax Laura is paying.

21. The length of a rectangle is twice its width. Write two equivalent expressions for the perimeter of the rectangle. Justify your response using properties of operations and the order of operations.

22. Show how you can use properties of operations as strategies to evaluate each expression using mental math. Then evaluate the expression.

- A. $0.2 + 7.9 + 3.8 + 1.1$
- B. $-5.6 + 5 + 4.6 + 1$
- C. $3\frac{1}{6} - 4\frac{3}{4} + 1\frac{5}{6}$
- D. $2\frac{1}{12} + 3\frac{1}{6} - 1\frac{1}{12}$

MATHEMATICAL PRACTICES

Reason Abstractly and Quantitatively

23. Nick is buying a birthday present for a friend. The gift is on sale for 40% off. He has a coupon for an additional 20% off the sale price. Write an expression to represent the cost of the gift. Then write your expression in another form to show what percent of the original price Nick will pay for his gift.

My Notes

MATH TERMS

An **equation** is a statement showing that two expressions are equal, such as $4 + 3 = 7$. An equation has an equal sign while an expression does not.

4. Use the expression you wrote for Melody's income to complete the table. Show your work.

Number of CDs Sold	Expression Used to Find Melody's Income	Melody's Income
10		
20		
100		

5. Assume Melody needs to make \$6,000 this month to cover her expenses. Write an equation you could use to find the number of CDs that Melody needs to sell to meet her expenses.

Check Your Understanding

6. Melody has hired a new accountant. He has gathered her pay stubs and is trying to determine how many CDs were sold during each month of the previous year. Her pay stub for June indicates that she made \$4,889 in that month. Write an equation her accountant could use to determine how many CDs were sold in June.
7. A photography studio charges a sitting fee of \$50 and \$10 per enlargement ordered. Write an equation to represent the number of enlargements ordered, n , if the total cost was \$180.
8. Does it seem reasonable that 18 enlargements were ordered in item 7? Explain.

LESSON 6-1 PRACTICE

9. The members of a Tae Kwon Do class are ordering jackets. Each jacket costs \$35, and there is a one-time fee of \$25 for the design. Write an equation to represent the number of jackets, n , that were ordered if the total cost is \$620.
10. A stockbroker charges his customers \$30 to open an account and \$15 per month to manage the account. Write an equation to represent the number of months, n , an account has been open if the total cost is \$360.
11. **Reason abstractly.** Lottie bought a new car for \$25,000. She paid \$5,000 up front and then \$600 per month. Write an equation to represent the number of months, n , it will take Lottie to pay for her car.
12. Mrs. Carter baked 100 muffins for a bake sale. The muffins were sold in packages of 2. There were 12 muffins left. Write an equation to find how many customers bought muffins at the bake sale.

Learning Targets:

- Solve two-step equations.
- Solve real-world problems by writing an equation of the form $px + q = r$.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Work Backward, Note Taking, Self Revision/Peer Revision

Work with your group to answer all parts of item 1. As you discuss your solutions, speak clearly and use precise mathematical language. Remember to use complete sentences and words such as *and*, *or*, *since*, *for example*, *therefore*, *because of* to make connections between your thoughts.

- Melody's friend Leena earns \$15 per hour as a lab technician plus an extra \$300 per week for singing at a club on the weekend.
 - Write an equation to represent the number of hours, n , Leena must work at the lab in a week to earn \$720.
 - Which number from the set {26, 28, 30} is the number of hours Leena must work?
 - How do you know that your answer to part b is correct?

Example A

Melody needs to record a new CD. She decides she can spend as much as \$8,000 on studio time. The studio charges \$425 to reserve the space and \$75 per hour. Solve the equation $75h + 425 = 8000$ to find the maximum number of hours Melody can afford to spend in the recording studio.

Step 1: Write an equation to represent the problem.

$$75h + 425 = 8000$$

Step 2: Use inverse operations. Subtract 425 from both sides.

$$75h + 425 - 425 = 8000 - 425$$

Step 3: Simplify both sides of the equation.

$$75h = 7575$$

Step 4: Use inverse operations. Divide both sides by 75.

$$\frac{75h}{75} = \frac{7575}{75}$$

Step 5: Simplify both sides of the equation.

$$1h = 101$$

Step 6: Use the Multiplicative Identity Property to isolate the variable.

$$h = 101$$

Solution: Melody can afford 101 hours of studio time.

My Notes

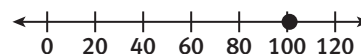
MATH TIP

Item 1 shows a problem solved algebraically using an equation with a variable. This problem can also be solved arithmetically. A possible arithmetic solution:

- Start with Leena's total earnings: \$720.
- Subtract her singing earnings (\$300) to find her lab technician earnings: $\$720 - \$300 = \$420$
- She earns \$15 per hour, so divide the lab technician earnings by \$15 to find the hours she must work: $\$420 \div \$15 = 28$ hours.

MATH TIP

You can graph the solution of an equation that has one variable on a number line. For the graph of the solution to Example A, the number line would have a dot at 101.



My Notes

Check the solution by substitution:

$$75(101) + 425 = 8000$$

$$7575 + 425 = 8000$$

$$8000 = 8000$$

The solution checks.

Try These A

Solve each equation below. Graph the solution to each equation on a number line.

a. $12n + 6 = 78$

b. $3x - 5 = 16$

c. $-8 = 2x + 4$

d. $2y - 3 = -9$

2. *Red Cast Records* pays a shipping company to ship CDs to retail stores. The shipping company is paid \$1,650 per shipment minus \$25 per hour for each hour the delivery arrives past the promised delivery time. The last shipment was late, so *Red Cast Records* was billed only \$1,325.
- a. Write an equation, using h to represent hours, that can be used to determine how late the delivery was made.
- b. **Make use of structure.** Use inverse operations to solve the equation algebraically. Check your solution.
- c. How late was the shipment?

So far, in this lesson and in the last lesson, you have written equations to represent problem situations expressed in words. It is also possible to work backwards; that is, write a problem situation in words that represents an equation.

3. Work with your group. Write a problem situation in words that represents each given equation. If you can, write situations that are related to music. Discuss how you will present your equations to the rest of the class. Remember to use words in your presentation that will help your classmates understand the situation.
- a. $8d - 250 = 750$

b. $12d + 400 = 1000$

DISCUSSION GROUP TIPS

As you discuss ideas for your equations and presentations, make notes and listen to what your group members have to contribute. Ask and answer questions to clearly aid comprehension and to ensure understanding of all group members' ideas.

Check Your Understanding

- 4. Which of the following values makes the equation $-9x + 17 = 8$ true?
A. -1 **B.** 0 **C.** 1 **D.** 3
- 5. **a.** A school reserved a banquet hall for the spring dance. In addition to a \$100 deposit, each couple must pay \$20. If the total cost of the banquet hall is \$1,140, write and solve an equation to find the number of couples attending the spring dance.
 - a.** Solve the problem arithmetically. Show the steps you used.
 - b.** Compare and contrast the steps you used to solve the problem algebraically and arithmetically.
- 6. Mia said that $2 - 3a = 11$ and $3a - 2 = 11$ have the same solution. Is she correct? Explain.
- 7. Solve each equation below algebraically.
 - a.** $5x - 2 = 13$
 - b.** $2a + 7 = -11$
 - c.** $\frac{1}{4}k + 3 = 6$
 - d.** $6 - 4a = -10$
 - e.** $15x = -15$

LESSON 6-2 PRACTICE

- 8. Solve each equation below algebraically.
 - a.** $6x - 11 = 19$
 - b.** $\frac{2}{3}y + 3 = 29$
 - c.** $8 - a = 17$
- 9. Solve and then graph each solution on a number line.
 - a.** $52 = 12 + 4w$
 - b.** $-24 = -6p$
- 10. The German Club is planning a ski trip. The club will pay \$500 toward the trip from money they have earned selling candy, and each member going on the trip will pay \$115. If the trip costs \$2,685, write and solve an equation to find the number of club members going on the trip.
- 11. **Make use of structure.** Explain the similarities and differences between guess and check and the algebraic method for solving an equation. Which method do you prefer? Why?
- 12. Write a problem in words that can represent this equation:
 $13x + 26 = 91$

My Notes

ACTIVITY 6 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 6-1

- Sam makes \$400 per week plus \$20 commission on each new cell phone plan she sells. Write an equation to determine how many new plans she sold to earn \$680 last week.
- Eric has a dog-walking business. He charges \$13 per dog he walks and \$6.00 for the water he buys for the dogs. If he made \$97 on Monday, write an equation to find the number of dogs he walked on Monday.
- In addition to dog walking, on Tuesday Eric made \$45 dog sitting at one of his customer's homes. If he made \$168 on Tuesday, write an equation to find the number of dogs he walked that day.
- Rena swims every day. She burns approximately 10.6 calories per minute when swimming and about 15 total calories warming up before she swims. Write an equation to find how many minutes Rena must swim to burn 205.8 calories.
- A cell phone company charges \$10 per month for up to 500 text messages and \$0.15 for each additional text message. Stanley was charged \$14.50 last month for text messages. Write an equation to find the number of text messages he sent and received last month.

Lesson 6-2

- Which of the following values makes the equation $4x - 8 = -4$ true?

A. -3	B. -1
C. 1	D. 3
- Solve each equation below algebraically.

a. $3x + 2 = 26$	b. $4c - 18 = 6$
c. $6a - 2 = 10$	d. $-7x + 3 = 17$
- Solve and graph the solution for each equation on a number line.

a. $3 + 4x = 11$	b. $8a - 6 = 18$
c. $\frac{1}{2x} + 3 = 9$	d. $17 = w - 4$

- Lacey and Chris solved the same equation, but their solutions were different. Who is correct? Justify your response.

<u>Lacey</u>	<u>Chris</u>
$20 + 8h = 180$	$20 + 8h = 180$
$20 - 20 + 8h = 180 - 20$	$\begin{array}{r} +20 \\ +20 \end{array}$
$8h = 160$	$8h = 200$
$\frac{8h}{8} = \frac{160}{8}$	$\frac{8h}{8} = \frac{200}{8}$
$h = 20$	$h = 25$

Jored and Sundai each want to buy a new HD movie player. They go to Electronics Superstore and find a HD movie player for \$75.00. Electronics Superstore offers different payment plans. Jored is going to pay \$15 now and then \$7.50 per month. Sundai is going to pay \$12.50 per month.

- Write and solve an equation for each plan to show how many months it will take each person to pay the \$75.00 for the HD movie player.
- It will cost \$285 to charter a bus for a class trip. The class treasurer says that there is \$60 in the class treasury. The 20 students going on the trip agree to make up the difference. Write and solve an equation to find how much each student will pay.
- A movie company sells DVDs on line. Each DVD costs \$9.95. No matter how many you order, the shipping charge is \$3.59. Your most recent order came to a total of \$33.44.
 - Write and solve an equation to find how many DVDs you ordered.
 - Solve the problem arithmetically. Show the steps you used.
 - Compare and contrast the steps you used to solve the problem algebraically and arithmetically.

MATHEMATICAL PRACTICES

Model with Mathematics

- Jason is given the equation $3x - 12 = 36$ to solve. His first step is to divide each term by 3. Do you think Jason's method is a good one to follow? Give an example to justify your answer.

Write your answers on notebook paper. Show your work.

1. Semir, Sarah, and SungSo decided to raise money for a local homeless shelter by working in a local deli. The deli agreed to donate to the shelter a portion of the profits from each meal the three sold. Semir sold 3 times as many meals as Sarah. SungSo sold 2 more meals than Sarah.
 - a. Write an expression for the number of meals that each sold.
 - b. If Sarah sold 24 meals, how many meals did Semir and SungSo each sell?
 - c. How many meals did the three sell in all? Explain how to use the commutative property and the associative property of addition to make finding the sum easier.
2. The three decided to participate in a walkathon to raise additional money for the shelter. Sarah walked 5 miles less than twice the number of miles that Semir walked. They each collected \$18 in pledges for every mile they walked.
 - a. Let n represent the number of miles Semir walked. Write an expression for the number of miles Sarah walked and an expression for the amount of money Sarah collected in pledges.
 - b. Write and solve an equation to find how many miles Semir walked if Sarah collected \$450 in pledges.
 - c. How many miles did Sarah walk?
 - d. SungSo also collected \$18 in pledges for every mile he walked. In addition, his grandmother gave him a \$72 donation. He collected the same amount of money as Sarah did. Write and solve an equation to find the number of miles SungSo walked.
3. Write a memo to the director of the shelter describing the total donation the three friends are sending and how it was raised. Be sure to include how much each student raised individually.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1a-c, 2a-d)	The solution demonstrates these characteristics:			
	<ul style="list-style-type: none"> A clear understanding of properties of operations. Effective understanding of and accuracy in writing and evaluating expressions and solving equations. 	<ul style="list-style-type: none"> A functional understanding of properties of operations. Writing and evaluating expressions, and solving equations that usually result in correct answers. 	<ul style="list-style-type: none"> Partial understanding of properties of operations. Difficulty with writing and evaluating expressions and solving equations. 	<ul style="list-style-type: none"> Little or no understanding of properties of operations. Little or no understanding of writing and evaluating expressions and solving equations.
Problem Solving (Items 1c, 2b, 2d)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. 	<ul style="list-style-type: none"> No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a, 2a-b, 2d)	<ul style="list-style-type: none"> Clear and accurate representations of problems as expressions and equations. 	<ul style="list-style-type: none"> Some difficulty in representing problems as expressions and equations. 	<ul style="list-style-type: none"> Difficulty in writing expressions and equations leading to errors. 	<ul style="list-style-type: none"> No understanding of representing problems as expressions and equations.
Reasoning and Communication (Items 1c, 3)	<ul style="list-style-type: none"> Precise use of appropriate math terms and language to explain solutions to problems and the role of properties of operations. 	<ul style="list-style-type: none"> Adequate explanation of solutions to problems and the role of properties of operations. 	<ul style="list-style-type: none"> Misleading or confusing explanation of solutions to problems and the role of properties of operations. 	<ul style="list-style-type: none"> Incomplete or inaccurate explanation of solutions to problems and the role of properties of operations.

Solving and Graphing Inequalities

It Plays to Save

Lesson 7-1 Modeling and Writing Two-Step Inequalities

Learning Targets:

- Represent quantities in a real-world problem.
- Construct two-step inequalities to solve problems.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Summarizing, Create Representations, Work Backward

Kerry’s grandmother, Mrs. Reynoso, is helping Kerry pay for guitar lessons. She has set up a special savings account to pay for the lessons. On the first of each month, the \$40 monthly fee for guitar lessons is withdrawn from the account.

1. Grandmother Reynoso started the account with \$300. Record the monthly balance in the account in the table.

Month	Balance
0	\$300
1	
2	
3	
4	

2. Write an expression for the amount of money in Kerry’s guitar lesson account. Let x equal the number of months since Mrs. Reynoso deposited the initial \$300.
3. Write an equation to determine when the account balance will be \$100.

The bank will close the account if the balance falls below \$50. To keep the account open, the balance must be greater than or equal to \$50.

4. Write an inequality to represent this situation.
5. Use the pattern in the table to find the last month the balance is \$50 or greater.

My Notes

My Notes

Inequalities can be used to represent many situations.

6. Think of a real life situation in which you might use terms such as *more than*, *less than*, *no more than*, or *no less than*.



7. a. Write the inequality represented by the graph above.
- b. Create a problem situation that could be represented by the graph above.
8. Arianna's mom deposits \$80 in her lunch money account. Lunch costs \$2.50 per day. Define a variable and write an inequality to represent when there will be less than \$20 left in Arianna's lunch account.
9. Michelle babysits on weekends in her neighborhood. She charges \$10 for transportation and \$15 for each hour she babysits. At her last babysitting job she earned less than \$60. Write an inequality to represent this situation.
10. Bailey can put x houses of his holiday miniature village on each of 7 shelves. He also has 10 houses displayed on his buffet. If he wants to display at least 45 of his houses, how many houses must he put on each of the 7 shelves? Write an inequality to represent this situation.

DISCUSSION GROUP TIPS

As you discuss ideas for your equations and presentations, make notes and listen to what your group members have to contribute. Ask and answer questions to clearly aid comprehension and to ensure understanding of all group members' ideas.

So far in this lesson, you have written inequalities to represent problem situations expressed in words. It is also possible work backwards; that is, write a problem situation in words that represents an inequality.

11. Work with your group. Write a problem situation in words that represents each given inequality. Remember to use real-life situations. Discuss how you will present your inequalities to the rest of the class. Remember to use words in your presentation that will help your classmates understand the situation.

a. $5x + 15 \geq 100$

b. $280 - 4m < 8$

My Notes

Learning Targets:

- Solve two-step inequalities.
- Construct two-step inequalities to solve problems.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Marking the Text, Summarizing, Create Representations, Think-Pair-Share

Consider the set $\{5, 7, 9\}$ and the inequality $6x - 8 < 46$.

1. Without solving the inequality, how can you determine which numbers from the set are solutions of the inequality?

2. Which numbers from the set are solutions?

3. Is 9 a solution? Why or why not?

Solving two-step inequalities is much like solving two-step equations. Use inverse operations to solve each of the following inequalities.

4. $2x - 10 < 80$
5. $5x - 8 + 7x > 40$
6. $7(x - 11) \leq 100$
7. $5x + 8.5 \geq -10.3$

There is one important difference, however, between solving equations and solving inequalities. The experiment below will help you discover this difference.

8. a. Work with a partner. Cut out the positive and negative number cards from the sheet your teacher will give you and stack the cards face down on your desk.
- b. Draw 2 cards. Write an inequality to represent the relationship between the numbers in the table below.
- c. Draw a third card. Multiply both sides of the inequality by the number indicated on this card. Record the result in the table. Is the result a true statement? If not, what can be done to make it a true statement?

Inequality Using First Two Numbers	Multiply on Both Sides By:	Inequality After Multiplication	True or False	Correction, If Necessary
$-3 < 5$	-2	$6 < -10$	False	$6 > -10$

Lesson 7-2

Solving Two-Step Inequalities

ACTIVITY 7

continued

- d. Based on your results and those of your classmates, what happens in an inequality when both sides of the inequality are multiplied by a negative number? What do you believe will happen when both sides of the inequality are divided by a negative number?

To solve a two-step inequality you isolate the variable just as you did when solving an equation. Remember to switch the inequality sign if you multiply or divide by a negative number.

Example A

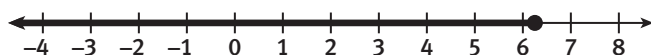
Solve $30 - 4x \geq 5$ and graph the solution on a number line.

Step 1: Original inequality $30 - 4x \geq 5$

Step 2: Subtract 30 from both sides. $30 - 30 - 4x \geq 5 - 30$
 $-4x \geq -25$

Step 3: Divide by -4 and reverse the inequality. $\frac{-4x}{-4} \leq \frac{-25}{-4}$

Solution: $x \leq 6.25$. The inequality symbol means less than or equal to, so 6.25 is part of the solution. This is shown by a filled-in circle on 6.25 on the graph of the solution.



Example B

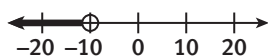
Solve $2x - 50 < -70$ and graph the solution on a number line.

Step 1: Original inequality. $2x - 50 < -70$

Step 2: Add 50 to both sides. $2x - 50 + 50 < -70 + 50$
 $2x < -20$

Step 3: Divide by 2. $\frac{2x}{2} < \frac{-20}{2}$

Solution: $x < -10$. The inequality symbol means less than, so -10 is not part of the solution. This is shown by an open circle on -10 on the graph of the solution.



My Notes

MATH TIP

Notice in Example B that you did not divide by a negative number so although the solution was negative, the inequality sign does not reverse.

My Notes

Try These A-B

Solve each inequality algebraically and graph your solution on a number line.

- $-5x + 7 > 22$
- $2x + 6 \geq 16$
- $-3(x + 5) < -21$
- $11x - 12 > 21$

- 9. Model with mathematics.** Hamid has read 60 pages of the book he will be using for a book report. If he reads 45 pages per hour, how many hours will it take him to read at least 375 pages of the book? Define a variable, and then write and solve an inequality to represent this situation. Graph the solution on a number line.

You should interpret the graph of the solution to an inequality in terms of the problem situation.

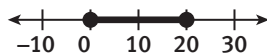
Example C

Joanne must complete a 4-question math quiz in 90 seconds or less. If she spends 30 seconds on the first question, what is the greatest amount of time on average she can spend on each of the remaining 3 questions? Write and solve an inequality to solve the problem. Then graph the solution and interpret the graph in the context of the problem.

Step 1: Write an inequality to represent the situation.
Let t represent the average amount of time in seconds for each of the remaining 3 questions.
She must complete the quiz in 90 seconds or less, so use \leq .
 $3t + 30 \leq 90$

Step 2: Solve.
 $3t + 30 \leq 90$
 $3t + 30 - 30 \leq 90 - 30$
 $3t \leq 60$
 $t \leq 20$

Step 3: Graph the solution.



Solution: Interpret the graph.
The graph shows that Joanne could spend an average of up to 20 seconds on each of the 3 remaining problems. Even though the graph is mathematically correct, it is very unlikely that she would spend any of the lower values, such as 0, 1, 2, 3, 4, and 5 seconds, on each problem.

Lesson 7-2

Solving Two-Step Equations

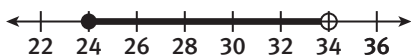
ACTIVITY 7

continued

Try These C

Interpret each graph.

- a. This graph shows the range in temperatures in degrees Fahrenheit during a day in February.



- b. This graph shows the height of water, in inches, in a rain barrel during a rainstorm.



My Notes

Check Your Understanding

- Solve each of the following inequalities algebraically. Graph the solutions on number lines.
 - $3x - 15 < 120$
 - $9x - 6 + 3x > 42$
 - $7(x - 1) \leq 35$
 - $-3x + 4 \geq -23$
- Falana has \$192 in her savings account. Since she is not using the account, the bank charges a monthly fee of \$5. The bank will close the account when the balance goes below \$50. Write and solve an inequality to find how many months until the bank closes Falana's account.
- Compare and contrast the solutions of $2x - 4 > 6$ and $2x - 4 = 6$.
- Write a short note to a friend explaining when to reverse the inequality sign when you are solving an inequality.

LESSON 7-2 PRACTICE

- Solve each of the following inequalities algebraically. Graph the solutions on number lines.
 - $2x - 50 > 75$
 - $15x + 20 > 50$
 - $-4x + 10 < 26$
 - $3x + 70 - 7x \geq 18$
 - $18 \leq -6x - 30$
 - $4(x + 2) > 60$
 - $2x + 6x - 9 < 39$
- Model with mathematics.** Arianna's mom deposits \$80 in her lunch money account. Lunch costs \$2.50 per day. Write and solve an inequality to determine when there will be less than \$20 in the account.
- Give three solutions of the inequality $5x - 2 > 7$.
- Consider the inequality $2x + 3 \leq -3$. Find the set of all integer solutions of this inequality that are also solutions of the inequality $5x - 2 < 3$.
- Compare and contrast solving equations and inequalities and their solutions.
- Nilsa is working on a 60-minute math test. There are 20 questions on the test. If it takes her 20 minutes to complete 12 of the questions, what is the greatest amount of time on average she can spend on each of the remaining 8 questions?
 - Write an inequality for the problem situation and solve it.
 - Graph the solution.
 - Interpret the graph in the context of the problem.

ACTIVITY 7 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 7-1

For 1–8, write an inequality to represent each situation.

- Twenty-two more than four times a number is less than 82.
- There are x giraffes at the zoo. The number of elephants is 4 less than three times the number of giraffes, and there are more than 23 elephants.
- Louie has 20 more than half as many baseball cards as Gerardo does. Together they have at least 350 cards.
- Zasha spent \$6 on packages of gum. How many more packages of gum that cost \$1.20 each can she buy if she has a \$20 bill?
- Dolores and four friends went to a buffet dinner. The total cost was at most \$130 including the \$20 tip they left. How much did each pay for the buffet?
- Eight less than five times the number of marbles that Iggy has is less than or equal to 72.
- George rented a bike for 4 hours. There was a \$10 deposit to pay in addition to the hourly rate. What was the hourly rate if the total came to less than or equal to \$65?
- A store wants to print flyers to advertise its grand opening. A printer will charge \$50 and \$0.05 per flyer. If the store has a budget of \$100, how many flyers can the store have printed without going beyond their budget?

Lesson 7-2

For 9–15, solve each inequality and graph the solution on a number line.

- $8x + 2 > 10$
- $12 - 2x < 16$
- $\frac{1}{2}x + 1 \geq 5$
- $2x + 7 - 3x \leq 10$
- Give three solutions of the inequality $6 - 11x < 61$.
- Which situation can be represented by the inequality $4x - 25 < 125$?
 - Frank bought four tires for x dollars each. He had a coupon for a \$25 discount. The total came to less than or equal to \$125.
 - Frank bought four tires for x dollars each. He paid \$25 in shipping for a total less than or equal to \$125.
 - Frank bought 25 tires for x dollars each. He paid \$4 in shipping for a total less than or equal to \$125.
 - Frank bought x tires for \$25 each. He paid \$4 in tax for a total less than or equal to \$125.

MATHEMATICAL PRACTICES**Reason Abstractly and Quantitatively**

- Carmine and Rachel went apple picking. Carmine has 5 more apples than Rachel. What is the minimum number of apples that Rachel has if there are at least 31 apples in all?

Write your answers on notebook paper. Show your work.

- The **media** reported that Olympic gold medalist Michael Phelps regularly consumed at least 8,000 calories per day when he was training for the Olympics. In order to do this, he ate at least three extra-large meals and also consumed a maximum of 2,000 calories worth of special energy drinks each day.
 - Write and graph an inequality to represent the number of calories from energy drinks Michael Phelps drank per day while training.
 - Write and graph an inequality to represent the total number of calories Michael Phelps consumed each day.
- Write a situation about Michael Phelps in which the inequality $3m + 2,000 \geq 8,000$ would represent the situation.
- To keep from losing weight while training, athletes must not burn more calories than they consume in a day. On one day, Michael Phelps burned 1,000 calories per hour while swimming and an additional 3,000 calories while out of the pool. Write and solve an inequality to estimate the number of hours he swam that day if his daily average caloric intake was at most 9,000 calories.
- An athlete wants to maintain a net caloric intake of no more than 2,000 calories for the day.
 - Write and solve an inequality to determine how many hours she must train if she burns an average of 750 calories per hour and eats a total of 8,000 calories.
 - Graph the solution to your inequality on a number line. Explain why your answer to part a is a solution to this situation.
 - If she trains 8 hours per day, what is the greatest caloric intake she can have to keep from losing weight? Explain your reasoning.

ACADEMIC VOCABULARY

Media are various ways by which news and information are communicated to the public. Media includes television, radio, and newspapers.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1a-b, 2, 3, 4a-c)	<ul style="list-style-type: none"> Effective understanding of and accuracy in writing, evaluating, and solving inequalities. 	<ul style="list-style-type: none"> Writing, evaluating, and solving inequalities correctly. 	<ul style="list-style-type: none"> Difficulty with writing, evaluating, and solving inequalities. 	<ul style="list-style-type: none"> Little or no understanding of writing, evaluating, and solving inequalities.
Problem Solving (Items 3, 4a, 4c)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. A correct and complete interpretation of the solution to an inequality. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer. A correct interpretation of the solution to an inequality. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. Difficulty interpreting the solution to an inequality. 	<ul style="list-style-type: none"> No clear strategy when solving problems. No understanding of interpreting an inequality or its solution.
Mathematical Modeling / Representations (Items 1a-b, 2, 3, 4a-b)	<ul style="list-style-type: none"> A clear and accurate representation of a situation as an inequality. Accurate and precise graphing of an inequality. 	<ul style="list-style-type: none"> Some difficulty in representing situations as inequalities. Correct graphing of an inequality. 	<ul style="list-style-type: none"> Difficulty in writing inequalities leading to errors. Some errors in graphing inequalities 	<ul style="list-style-type: none"> No understanding of representing situations as inequalities. Incomplete or inaccurate graphing of inequalities.
Reasoning and Communication (Items 2, 4b-c)	<ul style="list-style-type: none"> Precise use of appropriate math terms and language to explain solutions of inequalities. Clear and accurate writing of a situation to match an inequality. 	<ul style="list-style-type: none"> Adequate explanation of solutions to inequalities. Writing a situation to match an inequality. 	<ul style="list-style-type: none"> Misleading or confusing explanation of solutions to inequalities. Writing a situation that partially matches an inequality. 	<ul style="list-style-type: none"> Incomplete or inaccurate explanation of solutions to inequalities. An inaccurately written situation to match an inequality.